MASTER OF SCIENCE IN AEROSPACE ENGINEERING
PROPULSION AND COMBUSTION

COMBUSTION IN TWO-PHASE FLOW
Chap. 13 – COMBUSTION PHYSICS
R00 - 2011/11/27

LECTURE NOTES AVAILABLE ON
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Prof. Eng. Antonio Ficarella
University of Salento - antonio.ficarella@unisalento.it
DYNAMICS OF INDIVIDUAL DROPLETS

STATISTICAL CHARACTERIZATION OF SPRAY

COLLECTIVE INTERACTION OF DROPLETS WITH BULK GAS
GENERAL CONSIDERATION OF DROPLET COMBUSTION PHENOMENOLOGY

STRONG CONVECTION

flame blow-off – pure vaporization – combustion in the wake

Figure 13.1.1. Schematic showing droplet combustion configurations in the presence of externally imposed convection: (a) Weak convection that induces internal circulation; (b) strong convection, leading to separation and hence a complex recirculation pattern.
spherical flame

Figure 13.1.2. Photographic image of an almost spherically symmetric droplet combustion configuration, obtained for a suspended droplet in reduced pressure and hence buoyancy.
Figure 13.1.3. Particle track photographs. (a) Sphere without flame, $Re = 92$; (b) sphere with envelope flame, $Re = 92$; (c) sphere with wake flame, $Re = 152$ (Gollahalli & Brzustowski 1973).
flame image – soot emission
hydrocarbon flame – blue
droplet gasification rate
location and T of flame
extent of pollutant formation
droplets 10 – 100 μm
EXPERIMENTAL CONSIDERATIONS

- $d^2$-law combustion
  \[ r_s^2 = r_{s,0}^2 - K_c t \]

- Vaporization
  \[ r_s^2 = r_{s,0}^2 - K_b t \]
SINGLE-COMPONENTS DROPLET COMBUSTION
DROPLET HEATING

- transient droplet heating
- NO INTERNAL RECIRCULATION
  - heat transfer to droplet interior
  - effective heat latent of gasification

\[ m_{v}q_{v,\text{eff}} = m_{v}q_{v} + \left(4\pi r^{2} \lambda_{t} \frac{\partial T}{\partial r}\right)_{r=r^{-}}, \quad (13.2.1) \]

\[ \frac{\partial T}{\partial t} = \frac{1}{r^{2}} \frac{\partial}{\partial r} \left( \alpha_{h,\varepsilon} r^{2} \frac{\partial T}{\partial r} \right), \quad (13.2.2) \]

\[ T(r; t = 0) = T_{0}(r), \quad \left(\frac{\partial T}{\partial r}\right)_{r=0} = 0, \quad (13.2.3) \]
thermal diffusion coefficient

\[ \alpha_{h,t} = \frac{\lambda}{(c_p \rho e)} \]

unsteady conduction term
continuously regressing droplet surface \( r_s(t) \)
continuously changing surface temperature \( T_s(t) \)
droplet \( T \) assumed spatially uniform but temporally varying

\[ m_v q_v,\text{eff} = m_v q_v + \left( \frac{4}{3} \pi r_s^3 \rho e c_p \right) \frac{dT_s}{dt}, \quad (13.2.4) \]
(13.2.2) slowest limit – DIFFUSION LIMIT
(13.2.4) fastest limit (infinite conductivity) – DISTILLATION LIMIT
gasification Peclet n.

\[ Pe_h = \frac{K}{\alpha_{h,\ell}} \]

<1 distillation, >1 diffusion
CONVECTION (INTERNAL RECIRCULATION)

vortex core at $2/3 r_s$

$\bar{t} = \frac{\rho_g D_g}{(\rho l_{r_s,0}^2)t}$

gasification rate $m_b = \frac{m_b}{(4\pi \lambda/c r_s)}$

$R_s = \frac{r_s}{r_{s,0}}$

---

Figure 13.2.1. Temporal variation of the surface and center temperatures of an octane droplet after ignition, demonstrating the rapidity in the heating up of the droplet surface layer in both the diffusion and distillation limits.
Figure 13.2.2. Temporal variation of the burning rate constant and normalized effective latent heat of vaporization of the octane droplet of Figure 13.2.1, demonstrating: (a) the sequential nature of active droplet heating and gasification; and (b) active droplet heating spans only the initial period of the droplet lifetime.
Figure 13.2.3. Comparison of the size history of the octane droplet of Figure 13.2.1, predicted by various assumptions regarding droplet heating.
\( \frac{q_{b,\text{eff}} - q_b}{q_b} > 1 \) droplet heating

\( \frac{q_{b,\text{eff}} - q_b}{q_b} < 1 \) droplet gasification
FUEL VAPOR ACCUMULATION

- Fuel vapor accumulation between droplets and flame

\[ m_v = m_c + \frac{d}{dt} \int_{r_2(t)}^{r_f(t)} (Y_F \rho) 4\pi r^2 dr. \]  \hspace{1cm} (13.2.5)

- Gasification – combustion – ACCUMULATION/DEPLETION

Gasification rate at droplet surface \((m_v)\):

\[ = \text{Consumption rate at flame } (m_c) + \text{Accumulation/depletion rate in the inner region.} \]
reaction sheet standoff ratio

\[ \tilde{r}_f = \frac{\tilde{m}_c}{\ln (1 + \tilde{Y}_{O,\infty})} = 1 + \frac{\ln \left[ 1 + \frac{(T_f - T_s)/\tilde{q}_o}{\ln(1 + \tilde{Y}_{O,\infty})} \right]}{\ln(1 + \tilde{Y}_{O,\infty})}, \]  

(6.4.29)

Figure 13.2.4. Temporal variation of the flamefront standoff ratio, \( \tilde{r}_f \), for a heptane droplet burning in 300 K atmosphere, demonstrating that because of fuel vapor accumulation effects \( \tilde{r}_f \) continuously increases with time in low-\( \tilde{Y}_{O,\infty} \) environments but levels off in high-\( \tilde{Y}_{O,\infty} \) environments.
Figure 13.2.5. Temporal variation of the nondimensional flame radius, $R_f$, for the heptane droplet of Figure 13.2.4, demonstrating fuel vapor accumulation effects.

Figure 13.2.6. Temporal variation of the fractional fuel-consumption rate for the heptane droplet of Figure 13.2.4, demonstrating fuel vapor accumulation effects.
Figure 13.2.7. For the heptane droplet of Figure 13.2.4, demonstrating the concept that because of fuel vapor accumulation, the fractional amount of fuel reacted at any instant is always less than the fractional amount of fuel gasified.
\[
\frac{m_c}{4\pi r_s} = \ln \left\{ \left[ 1 + \frac{c_{p,1}(T_f - T_s)}{q_v} \right]^{(\lambda_1/c_{p,1})} \left( 1 + Y_{\Theta,\infty} \right)^{(\rho D)} \right\}
\]

\[
\bar{r}_f = 1 + \frac{(\lambda_1/c_{p,1})}{(\rho D)} \ln \left[ 1 + c_{p,1}(T_f - T_s)/q_v \right]
\]

\[
q_c = (c_{p,2} T_f - c_{p,1} T_s + q_v) + \frac{c_{p,2}(T_f - T_\infty)}{\left[ (1 + Y_{\Theta,\infty})^{1/L_{eff}} - 1 \right]}
\]

\[
L_{eff} = \frac{\lambda_1/c_{p,1}}{(\rho D)}
\]
GAS-PHASE TRANSIENT DIFFUSION AND HIGH PRESSURE COMBUSTION

- two major processes
  - droplet heating
  - fuel accumulation
- limit of quasi-steady assumptions flow-velocity show such that diffusion-time same order of surface regression

\[
\frac{r_{\infty}^2}{D_g} \sim \frac{r_s^2}{K}.
\]

\[
\frac{r_{\infty}}{r_s} \sim \left(\frac{\rho_t}{\rho_g}\right)^{1/2}.
\]

since
Figure 13.2.8. Calculated droplet size history with various extents of gas-phase transient diffusion as represented by $\delta \sim [\bar{m}(\rho_g/\rho_e)]^{1/2}$. 
transient diffusion decrease droplet gasification rate
important when in critical and supercritical conditions
CRITICAL PRESSURE
reduced latent heat vaporization – LIFETIME DECREASES
mass diffusivity effects – mixing between vapor phase and ambient gas
Figure 13.2.9. Experimentally determined droplet burning lifetime as a function of pressure under normal gravity and microgravity conditions (Sato et al. 1990).
CONVECTION EFFECTS AND DROPLETS DYNAMICS

- vigorous burning – external convection increases droplet gasification rate

\[
\frac{K}{K^o} = 1 + \frac{0.276 Re^{1/2} Pr^{1/3}}{(1 + 1.232/Re Pr^{4/3})^{1/2}}, \quad (13.2.13)
\]

- GRASHOF – buoyancy/viscous

\[
\frac{K}{K^o} = 1 + 0.52 Gr^{1/2}, \quad Gr < O(1) \quad (13.2.14a)
\]
\[
= 1 + 0.85 Gr^{1/4}, \quad Gr > O(10), \quad (13.2.14b)
\]
Figure 13.2.10. Dependence of the normalized burning rate constant, $K/K^0$, on the Grashof number, showing variation of the Grashof number scaling.
- high convection intensity – flame blow-off
- vaporization combustion – drag of droplet

Figure 13.2.11. Comparison of the calculated drag coefficient with the standard drag curve for vaporizing droplets, demonstrating the approximate cancellation of the pressure drag and friction drag such that the standard drag curve can still be used (Reeksizbulut & Yuen 1983).
CIRCULARITY MOTION INSIDE THE DROPLET

however – burning time reduced

significant concentration non-uniformity
DROPLETS INTERACTION

- SPRAY
- interaction reduces gasification rate
  - competition for oxygen
- $d^2$-law does not hold
Figure 13.2.12. Burning sequence of a three-droplet array with (a) minimum and (b) strong buoyancy. The center droplet in (a) has a larger flame because it has less oxygen supply; it burns out last while the center droplet in (b) burns out first.
burning rate $K_c(t)/K_{c,\text{isolated}}$

separation distance $L/d_s(t)$
DROPLET COLLISION

- Weber number, $\text{We} = d_s \rho_l U^2 / \sigma$
- Impact parameter, $B = X / d_s$

Figure 13.2.14. Collision regime diagram for: (a) hydrocarbon droplets at atmospheric pressure and water droplet at elevated pressures, (b) water droplet at atmospheric pressure and hydrocarbon droplets at reduced pressures.
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<th>Regime I</th>
<th>Time (ms)</th>
<th>Regime II</th>
<th>Time (ms)</th>
<th>Regime III</th>
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Figure 13.2.15. Photographic images showing representative head-on collision in Regimes I–IV.
Figure 13.2.16. Photographic images showing representative off-center collision in Regimes I, II, III, and V.
HEAD-ON COLLISION

- regime I
- negligible kinetic energy
- energy $\rightarrow$ surface tension
- energy large diameter $<$ small diameter
- conversion surface energy $-$ kinetic energy $\rightarrow$ oscillations $\rightarrow$ viscous dissipation

INCREASE OF ENERGY

- regime II $-$ III $-$ IV
- large collision energy

OFF-CENTER COLLISION

- rotation motion
- regime V
IGNITION AND EXTINCTION CRITERIA

mass burning rate

\[
\frac{\bar{m} \, d T}{\bar{r}^2 \, d \bar{r}} - \frac{1}{\bar{r}^2} \frac{d}{d \bar{r}} \left( \bar{r}^2 \frac{d T}{d \bar{r}} \right) = D_{ac} Y_O Y_e e^{-\frac{T_u}{T}}. \tag{13.2.20}
\]
SYNTHETIC FUELS
- more complex composition
- wider boiling point range
- water – oil emulsions
- coal – water slurries
- concentration and velocity of constituents
- miscibility
- diffusion, surface regression, motion within the droplet
- rate with which liquid components can be brought to the surface
MISCIBLE MIXTURES

- exposure
  - PASSIVE – surface regression
  - ACTIVE – diffusion – internal circulation

\[
\frac{\partial Y_{\ell,t}}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( D_\ell r^2 \frac{\partial Y_{\ell,t}}{\partial r} \right).
\]  

(13.3.2)

- Pe\textsubscript{m} mass diffusion
- Pe\textsubscript{h} heating
- frozen composition – onion model Pe\textsubscript{m} → 0 D\textsubscript{l} → 0
- uniformed composition – volatility controlled – batch distillation Pe\textsubscript{m} → ∞ K → 0
hydrocarbons

\[ K = 10^{-3} - 10^{-2} \text{ cm}^2/\text{s} \]
\[ D_l = 10^{-5} - 10^{-4} \text{ cm}^2/\text{s} \]
\[ \text{Pe}^h = 1 \]
\[ \text{Le}^l = \text{Pe}_{m}^{-1} = 10 - 10^3 >> 1 \]

diffusion limited combustion
Figure 13.3.1. Development of the concentration profiles within the droplet at different stages of the droplet lifetime, $M = 1 - R_s^2$, showing establishment of the concentration boundary layer at the surface and its persistence throughout most of the droplet lifetime.
Figure 13.3.2. Concentration history of the volatile component at the surface and center locations of the droplet demonstrating, for large $Le$, the initial transition involving changes in the surface layer, a subsequent nearly steady-state behavior, and a final, extremely short period of volatility-dominated gasification. With decreasing $Le$ and, hence, less diffusional resistance, the transitions become less distinct.
Figure 13.3.3. Temperature history for the same case as Figure 13.3.2.
Figure 13.3.4. Experimental $d_i^2$-$t$ plot for a heptane-propanol droplet. The nearly constant regression rate of each individual mixture indicates that the droplet gasifies like a pure component one because the volatilities of its components are almost the same.

Figure 13.3.5. Experimental $d_i^2$-$t$ plot for a heptane-hexadecane droplet, demonstrating the three-stage behavior during the gasification of a two-component droplet with vastly different volatilities.
- flame size + gasification rate
- flame size will decrease during gasification of less-volatile components (2\textsuperscript{nd} stage)

Figure 13.3.6. Flame streak of a freely falling burning droplet stream, showing the phenomenon of flame shrinkage due to transitional droplet heating as the droplet surface layer becomes more concentrated with the less volatile component.

Figure 13.3.7. Flame size variations demonstrating the phenomenon of flame shrinkage.
Figure 13.3.8. Experimental result on the temporal variation of the molar fraction of the more volatile component in a two-component alkane droplet, demonstrating the weakening of diffusional resistance with decreasing liquid-phase Lewis number.
MICRO-EXPLOSION PHENOMENON

- volatile component in the inner core
- droplet temperature attains a high value – controlled by the high-boiling component at the surface
- the liquid element in droplet interior will nucleate and gasify

Figure 13.3.9. Flame streak of a freely falling droplet stream, showing the phenomenon of droplet microexplosion.
Figure 13.3.10. Schematic showing the gasification and shell formation mechanism of a slurry droplet.
limit of super-heat = 90% critical
  \[ T(r,t) > T_L[X_{i,l}(r,t)] \]

micro-explosion facilitated with increasing pressure
  elevation of boiling T at surface
  \[ T_L \text{ constant } \neq f(p) \]

opt. composition 50% - 50%

MICROEXPLOSION
  opt. droplet size distribution
  utilization of synthetic, less refined fuels
EMULSION AND SLURRIES

- Unlike a miscible mixtures → MULTIPHASE MIXTURES
- Water-in-oil emulsions
  - Reduction in soot and NOx
  - Increase in CO HC
- Micro-explosion with great intensity
- Emulsion of water with heavy oil or jet fuel
  - Difficult to ignite
  - Difficult steady burning
- Coal-oil (slurry)
- Coal-water (slurry)
  - Micro-explosions
  - Solid particles facilitate nucleation
ALCOHOLS AND REACTIVE LIQUID PROPELLANTS

- heat source at the droplet surface of its interior
  - droplet gasification can be facilitated
water condensation

- boiling point of ethanol – methanol < water
- condensation heat release
- $K_b$ subsequently decreases because of higher water content

Figure 13.3.11. Experimental result on the temporal variation of the vaporization rate constant of a methanol droplet in dry and humid environments, demonstrating that water vapor condensation at the droplet surface can initially increase the gasification rate through the condensation heat release, but will eventually slow down the gasification rate as the droplet becomes highly concentrated with water.
Figure 13.3.12. Experimental result on the temporal variation of the water content in the droplet for burning methanol (MeOH) and ethanol (EtOH) droplets, demonstrating that flame-generated water vapor can condense at the droplet surface and subsequently dissolve into the droplet interior.
CARBON PARTICLE COMBUSTION PHENOMENOLOGY

SEQUENCE OF EVENTS

PARTICLE LIFETIME 10%
- drying
- pyrolysis of volatile matter
- emission of fuel vapor
- ignition
- envelope of wake flame

PARTICLE LIFETIME 90%
- solid pyrolysis – remaining particle coke or char - porous
- burning of the char carbon gasification
- mineral oxides left behind
boiling point of liquid fuels – 700-800 K
carbon sublimation 4000 K
particle T – radiative loss – 2500-3000 K
low carbon burning rate
  surface reaction important
GLOBAL KINETICS OF CARBON OXIDATION

\[
\begin{align*}
C + O_2 & \rightarrow CO_2 \\
2C + O_2 & \rightarrow 2CO \\
C + CO_2 & \rightarrow 2CO \\
C + H_2O & \rightarrow CO + H_2.
\end{align*}
\]

- C2 preferred at higher T > 1000 K
- C3 higher T > 1600 K
- C5 water catalyzed

\[
2CO + O_2 \rightarrow 2CO_2.
\]
ANALYSIS

- surface reactions C2 – C3
- gas-phase C5
- O2 reaches the particle surface → CO₂ reaction
- as the particle heats up → fig. 13.4.1b

Figure 13.4.1. Possible burning configurations of carbon: (a) frozen gas-phase reaction, (b) detached flame-sheet burning.
\[
L(Y_{CO} + Y_{CO_2}) = L(Y_{O_2} + Y_{CO_2}) = L(Y_{CO_2} - T) = 0 \quad (13.4.1)
\]
\[
L(T) = \dot{w}_g, \quad (13.4.2)
\]
\[
L(\cdot) = \left[ \frac{m}{\tilde{r}^2} \frac{d}{d\tilde{r}} \left( \frac{r}{\tilde{r}^2} \frac{d}{d\tilde{r}} \right) \right] (\cdot)
\]
\[
\tilde{r} = 1: \quad T = T_s, \quad Y_i = Y_{i,s}, \quad i = CO, O_2, CO_2
\]
\[
\tilde{r} = \infty: \quad T = T_\infty, \quad Y_i = Y_{i,\infty}, \quad Y_{CO} = 0, \quad i = O_2, CO_2, \quad (13.4.3)
\]
\[
mY_{CO,s} - 4\pi r^2 \rho D \left( \frac{dY_{CO}}{dr} \right)_{r_z} = m_{CO}. \quad (13.4.4)
\]
\[
\dot{m}Y_{CO,s} - \left( \frac{dY_{CO}}{d\tilde{r}} \right)_1 = \dot{m}_{C,1} + 2\dot{m}_{C,2}, \quad (13.4.5)
\]
\[
\dot{m}Y_{O_2,s} - \left( \frac{dY_{O_2}}{d\tilde{r}} \right)_1 = -\dot{m}_{C,1}, \quad (13.4.6)
\]
\[
\dot{m}Y_{CO_2,s} - \left( \frac{dY_{CO_2}}{d\tilde{r}} \right)_1 = -\dot{m}_{C,2}. \quad (13.4.7)
\]
LIMITING SOLUTIONS

- Gas-phase reaction
  - Frozen
  - Infinitely fast – detached flame
- Surface reactions proceed at finite rate
- Attached flame – $\bar{m}$ decreasing
- Diffusion limit $\rightarrow$ $d^2$-law
- Surface reaction limit $\rightarrow$ $d$-law
  - $dr_s/dt$ cons
  - Solid particles without pores – reactions over surface of pores
- Small burning rate limit
- Intricate couplings
Figure 13.4.2. Limiting solutions of carbon particle combustion, bracketing all feasible combustion states ($Br_x/D_\infty = 10^8$, $Y_{CO_2,\infty} = 0$).
METAL PARTICLE COMBUSTION

- combustion products are solid
- condensed phase of oxides at the particle surface (OXIDE COATINGS)
- complex metal – oxide system
PHENOMENOLOGY OF SPRAY COMBUSTION
ONE-DIMENSIONAL PLANAR SPRAY FLAME

- droplet vaporization
- premixed burning (stoichiometric) – heat release region
- droplet burning

Figure 13.6.1. Schematic of one-dimensional spray combustion.
min. size below which droplet burning not considered
small droplet will be completely vaporized in the pre-heat zone of flame
classic time for traversing the pre-heat zone

dequation

\[ \tau_D = \frac{L_D^0}{s_\alpha^0} = \frac{\lambda/c_p \rho_k}{(s_\alpha^0)^2}, \quad (13.6.1) \]

droplet vaporization time

dequation

\[ \tau_v = \frac{d_{s.o}^2}{4K_v} = \frac{d_{s.o}^2 \rho_c}{8(\lambda/c_p) \ln(1 + B_v)}, \quad (13.6.2) \]
\[ d_{s, \text{min}}^2 = \frac{8 \rho_l}{\rho_\ell} \left( \frac{\lambda/c_p}{f_0} \right)^2 \ln(1 + B_v), \] (13.6.3)

\[ \approx p^{(1-n)} \approx 10 \mu m \]

\[ F/A \approx \frac{1}{6} \pi \frac{d_f^3 \rho_\ell}{\rho_g} = \frac{\rho_\ell}{\rho_g} \left( \frac{d_f}{d_g} \right)^3, \] (13.6.4)

\[ F/A \approx 0.05 \ d_g/d_s \approx 25 \ (p = 1 \text{ bar}) \ d_g/d_s \approx 15 \ (p = 15 \text{ bar}) \]

\[ d_f/d_s \approx 5 - 10 \]
SPRAY JET FLAMES

- oxidizing gas within the spray interior
- rate of droplet vaporization
- rate of oxidizer entrainment
- droplet combustion for entrainment rate
- jet sheath combustion

Figure 13.6.2. Schematic of spray jet combustion.
CLOUD/DENSE SPRAY COMBUSTION

- droplets close to each other
- spray interior oxidizer lean
- cloud combustion
- group combustion
- group combustion n. $G \approx n l^3$
  - $n$ droplet number density
  - $l$ characteristic dimension of spray
(a) External group combustion with “sheath vaporization” at high $G$.  
(b) External group combustion with a “standoff” flame.  

(c) Internal group combustion with the main flame located within the spray boundary.  
(d) Single droplet combustion at low $G$.  

Figure 13.6.3. Schematic showing the four group combustion modes of a spherical droplet cloud with decreasing $G$ from (a) to (d) (Chiu, Kim & Croke 1983).
Figure 13.6.4. Schematic showing the group combustion concept of a spray jet (Chiu et al. 1978).
SPRAY STATISTICS
SPRAY DISTRIBUTION FUNCTION

- Spray distribution function at time $t$
  
  \[ f(r_s, x, u, t) \, dr_s \, dx \, du \]

- Deviate from spherical symmetry

\[ \text{We} = 2r_s \rho g |u - v|^2 / \sigma, \]

- $\text{We} < 20$ droplets deform and break-up
- Strong surface tension promote the sphericity
  - Solid particles
SPRAY EQUATION

\[
\frac{\partial f}{\partial t} + \frac{\partial}{\partial r_x} (\dot{r}_x f) + \nabla_x \cdot (u f) + \nabla_u \cdot (g f) = S,
\]  

(13.7.1)
DROPLET SIZE DISTRIBUTION FUNCTION

- \( G(r_s) \)
- Number and volume of droplets \( r < r_s \)

\[
\int_0^{r_s} G(r_s) \, dr_s \quad \text{and} \quad \frac{4\pi}{3} \int_0^{r_s} r_s^3 G(r_s) \, dr_s.
\]

SAUTER MEAN DIAMETER

\[
2 \sum \frac{n_i(r_{s,i}^3)}{n_i(r_{s,i}^2)} \quad \text{and} \quad 2 \frac{\int_0^{r_{s,\text{ext}}} r_s^3 G(r_s) \, dr_s}{\int_0^{r_{s,\text{ext}}} r_s^2 G(r_s) \, dr_s}
\]

(13.7.2)

ROSIN-RAMMLER DISTRIBUTION

- \( t = s - 4 \) Rosin-Rammler
- \( s = 1 \) chi-square

\[
G(r_s) = b(r_s)^t \exp[-a(r_s)^t].
\]
MONO-MODAL SIZE DISTRIBUTION

\[ n = \int_0^\infty Gdr_s, \quad <r_s> = \frac{1}{n} \int_0^\infty r_s Gdr_s, \]

\[ \sigma = \frac{1}{n} \int_0^\infty (r_s - <r_s>)^2 Gdr_s, \quad s\sigma^3 = \frac{1}{n} \int_0^\infty (r_s - <r_s>)^3 Gdr_s. \]

skewness (asymmetry)
CONSERVATIVE EQUATIONS

- fluid density
- gas density
- continuity of gas

\[\frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \mathbf{v}) = - \int \int \rho_c (4\pi r_s^3) \dot{r}_s f dr_s d\mathbf{u}. \]  \hspace{1cm} (13.7.4)

- conservation of momentum
  - \textbf{g} force exerted on the droplet by flue gas
  - momentum transferred by vaporizing mass

\[\rho_f \frac{\partial \mathbf{v}}{\partial t} + \rho_f \mathbf{v} \cdot \nabla \mathbf{v} = - (\nabla \cdot \mathbf{P}) + \rho_f \sum_{i=1}^{N} Y_i f_i - \int \int \rho_c \left(\frac{4}{3} \pi r_s^3\right) \mathbf{g} f dr_s d\mathbf{u} + \int \int \rho_c (4\pi r_s^2) \dot{r}_s (\mathbf{u} - \mathbf{v}) f dr_s d\mathbf{u}, \]  \hspace{1cm} (13.7.6)
conservation of energy

\[
\frac{\partial}{\partial t} \left[ \rho_f \left( h_f + \frac{v^2}{2} \right) \right] + \nabla \cdot \left[ \rho_f \mathbf{v} \left( h_f + \frac{v^2}{2} \right) \right] = -\nabla \cdot \mathbf{q} - \mathbf{P} : (\nabla \mathbf{v}) + \frac{\partial p}{\partial t} + \rho_f \sum_{i=1}^{N} Y_i \mathbf{f}_i \cdot (\mathbf{v} + \mathbf{V}_i) - \int \int \rho_f \left( \frac{4}{3} \pi r_s^3 \right) (\mathbf{g} \cdot \mathbf{u}) f dr_s du - \int \int \rho_f \left( 4 \pi r_s^2 \right) \dot{r}_s \left( h + \frac{u^2}{2} \right) f dr_s du.
\]  

(13.7.8)

conservation of species

\[
\frac{\partial}{\partial t} \left( \rho_f Y_i \right) + \nabla \cdot \left[ \rho_f (\mathbf{v} + \mathbf{V}_i) Y_i \right] = w_i - \int \int \rho_f \left( 4 \pi r_s^2 \right) \dot{r}_s \Omega_i f dr_s du.
\]  

(13.7.9)
ADIABATIC SPRAY VAPORIZATION

- isolated droplet
- vaporization rate
  \[
  \dot{m} = m/[4\pi(\lambda/c_p)r_s]
  \]
  \[
  \dot{m} = \ln(1 + B_0)
  \]  
  \[\text{(13.8.1)}\]
- vapor concentration at surface
  \[
  Y_s = \frac{B_v + Y_\ell}{1 + B_0},
  \]  
  \[\text{(13.8.2)}\]
- vapor mass flux
  \[
  B_{h,v} = \frac{c_p(T_g - T_s)}{a_v + a_\ell}
  \]  
  \[\text{(13.8.3)}\]
- energy balance
  \[
  (1 + \eta Y_{\ell,o})c_p T_g + (1 - \eta)Y_{\ell,o}(c_p T_s - q_v) = c_p T_{g,o} + Y_{\ell,o}(c_p T_{s,o} - q_v)
  \]  
  \[\text{(13.8.4)}\]
- mass conservation
  \[
  \eta = 1 - R_s^3
  \]
  \[
  Y_g = \frac{Y_{g,o} + \eta Y_{\ell,o}}{1 + \eta Y_{\ell,o}},
  \]  
  \[
  R_s = \frac{r_s}{r_{s,o}},
  \]  
  \[\text{(13.8.5)}\]
vaporization occurs $T_g > T_s \ Y_g > Y_s$

if droplet attains an equilibrium temperature $T_{s,l}$

min. chamber length to achieve complete spray vaporization

$x^* \approx r_{s,0}^2$

$$x^* = \frac{u_0 r_{s,0}^2 \rho \ell}{2(\lambda/c_p) \bar{m}_e \left(1 + 0.6 \frac{Y_{t,o}}{\bar{m}_e}\right)}.$$  \hspace{1cm} (13.8.24)
HETEROGENEOUS LAMINAR FLAMES

- premixed gaseous combustible
  - laminar flame speed $s_u$
- two-phase mixtures
  - homogeneous burning <$10\mu$m
  - non-premixed burning around the droplets – $d^2$-law
- $\sigma$ oxidizer-to-fuel mass ratio
- $(1-\sigma)$ mass of mixture reacted/mass of fuel burned

$$w \approx (1 + \sigma) \int_0^\infty \rho_\ell \left(4\pi r_s^2\right) (-\dot{r}_s)Gdr_s.$$  \hspace{1cm} (13.9.1)

$$\dot{r}_s = \frac{dr_s}{dt} = \frac{1}{2r_s} \frac{dr_s^2}{dt} = -\frac{K_c/2}{r_s}.$$ \hspace{1cm} (13.9.2)

$$K_c = 2[(\lambda/c_p)/\rho_\ell] \ln(1 + B_{h,c})$$
uniform droplet size

\[ w \approx 2\pi (1 + \sigma)n_0 r_s \rho \ell K_c, \quad (13.9.3) \]

laminar burning flux

\[ f^2 = (\rho_0 u_0)^2 = (\lambda/c_p)w, \quad r_s \rightarrow r_{s,0} \]

\[ f^2 = 2\pi (1 + \sigma)n_0 r_{s,0} \rho \ell (\lambda/c_p)K_c \quad (13.9.4a) \]

\[ = 4\pi (1 + \sigma)n_0 r_{s,0}(\lambda/c_p)^2 \ln(1 + B_{h,c}). \quad (13.9.4b) \]

initial mass of liquid

\[ n_0 \rho \ell \left( \frac{4}{3} \pi r_{s,0}^3 \right) = \text{constant} \quad n_0 \sim (r_{s,0})^{-3}, \]

\[ f \sim \frac{\lambda/c_p}{r_{s,0}}. \quad (13.9.5) \]
GAS-PHASE FLAMES

\[ f = \rho u = \rho_0 u_0, \quad (13.9.6) \]

\[ nu = n_0 u_0, \quad (13.9.7) \]

**Figure 13.9.1.** Schematic showing the dependence of the burning velocity on the initial droplet size (- heterogeneous limit; --- two possible transitional behavior).

- effect of \( d^2 \)-law
- as \( r_{s,0} \to 0 \) homogeneous burning + latent eat of vaporization
CONDENSED-PHASE FLAME

- diffusion-controlled heterogeneous flame
- exhibit as premixed flame

Figure 13.9.2. Schematic showing flame structure in the self-propagating high-temperature synthesis process (SHS).